THE DEVELOPMENT OF BINARY ARITHMETIC BY LEIBNIZ: INFLUENCE OR INDEPENDENCE REGARDING THE XIANTIANTU OF SHAO YONG

Marie-Julie Maitre*

Abstract: The French Jesuit Joachim Bouvet studied the Xiantiantu, a diagram of Shao Yong that organizes the hexagrams according to the "Fuxi order." While exchanging letters with Leibniz, Bouvet discovered an analogy between his binary arithmetic and the Xiantiantu. It indicates that the Xiantiantu could be considered binary, and it has implications in the history of binary arithmetic. Consequently, it would be essential to analyze a persistent idea that Leibniz developed binary arithmetic while being influenced by the Xiantiantu, which is opposed to the usual historical chronology. This article investigates whether Leibniz developed his calculation independently or under the influence of Shao Yong's diagram by introducing an original argument in the discussion. It will be shown that the Xiantiantu did not influence Leibniz because if this was the case, it would have totally changed his view on Chinese writing and, consequently, his research on Universal characteristics.

Introduction[1]

The French Jesuit Joachim Bouvet, Bai jin (1656-1730) studied the *Yijing* or *Zhouyi*, the *Book of Changes*, one of the five Chinese Classics, which is often described as "a system of notation of acts of divination" (Cheng, 1997 (2002), 268). However, it is more of a complete enumeration of interpretations of all the sixty-four mantic figures composed of eight broken or continuous lines that can result from a specific divination technique (usually a procedure of counting off yarrow stalks). Notations are represented as eight figures, or trigrams made up of three solid lines (yang 陽) and broken lines (yin 陰).[2] The drawing of six stems of yarrow stalks gave six superimposed lines that formed a hexagram. The set of combinations of the six lines makes up sixty-four hexagrams. Bouvet especially focused on the *Xiantiantu* (Fig. 1),

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[1] Unless noted, all translations are mine, and the author refers to the original by space limitation. For the Leibniz-Bouvet correspondence, The author quotes the translation of Daniel J. Cook and Alan Berkowitz, *Correspondance Leibniz-Bouvet* (hereafter CB); I refer to the original in the Gottfried Wilhelm Leibniz, *Sämtliche Schriften und Briefe* (hereafter A); as well as in Widmaier and Babin (2006) (hereafter WB).

[2] The yin and the yang are the fundamental symbols of Chinese thought and cosmogony, which express the complementary opposites and constitute a unity. The yin expresses flexibility, fertility, and femininity. It used to represent the shadowy side of a mountain. The yang expressed the idea of activity, dynamism, and masculinity and represented the sunny side of the mountain. They are dynamic principles and movements that complement each other and are necessary for the world because everything is made up of yin and yang.

a diagram of Shao Yong (1011-1077)[3] that he attributed to the mythical emperor Fuxi (2900 BCE).[4] This diagram is an organized system of the *bagua* that formed the sixty-four hexagrams according to the Fuxi or the *Xiantian* order (Fig. 2). It differs from eight trigrams (Fig. 3) and the sixty-four hexagrams (Fig. 4) according to the Wen Wang or *Houtian* order.

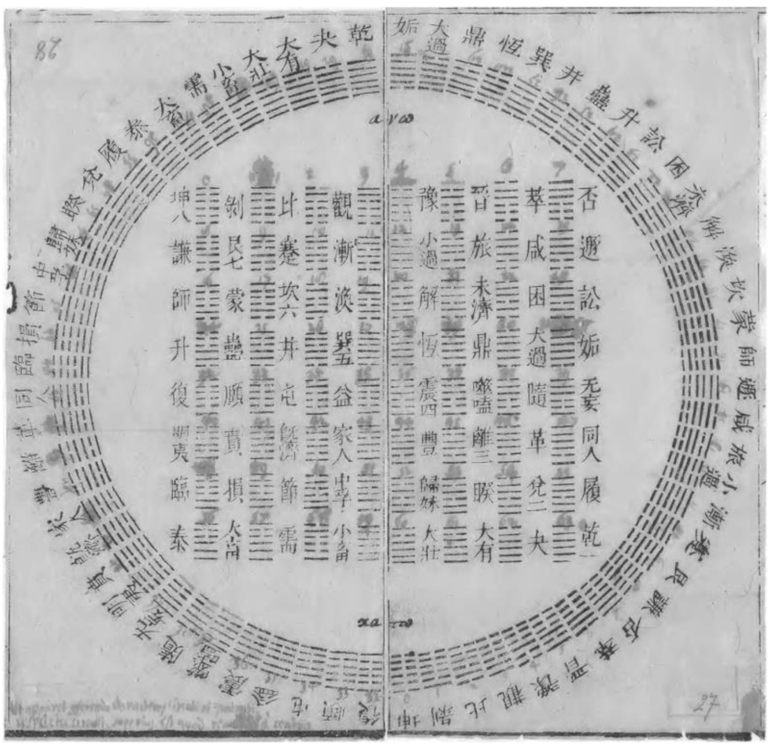
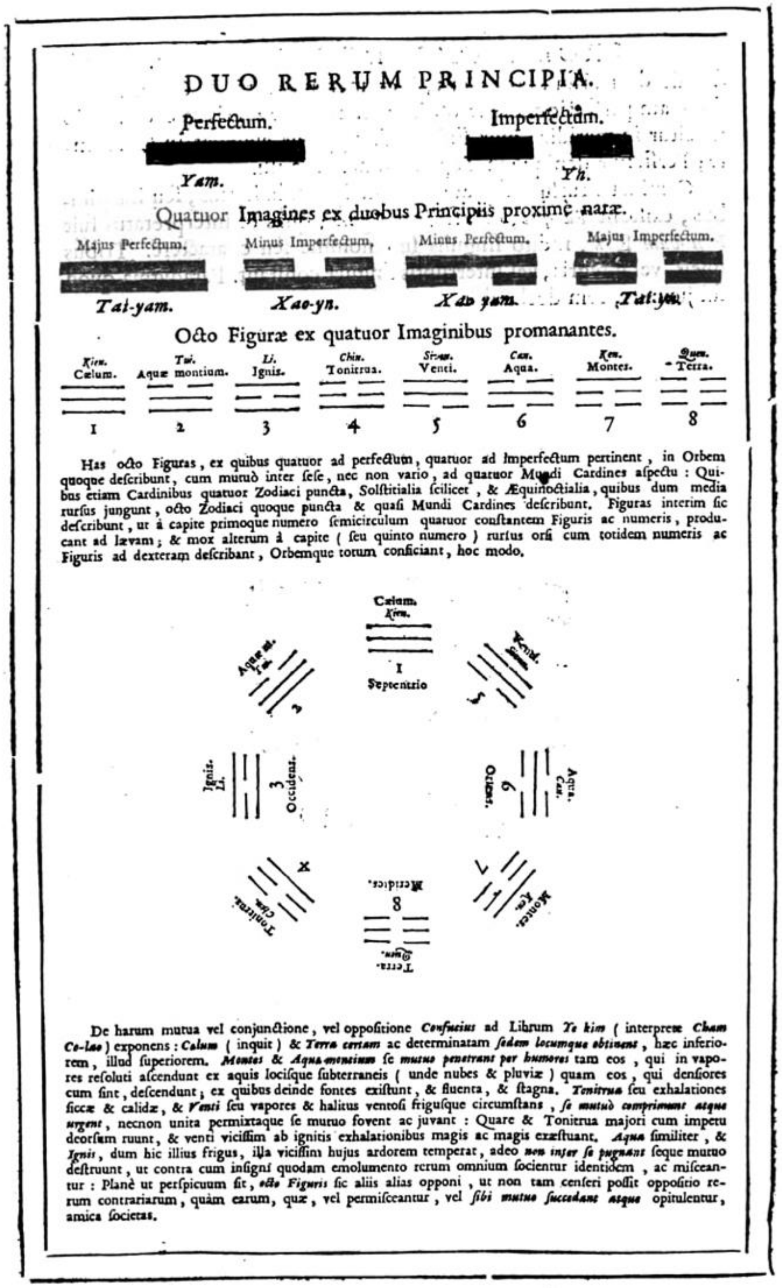


Figure 1- Diagram of the *Xiantiantu* sent by Bouvet to Leibniz with his letter of 4 November 1701 (A I, 20, 556). Source: Gottfried Wilhelm Leibniz Bibliothek – Niedersächsische Landesbibliothek, Hannover (shelf mark: LBr 105, Bl. 27(LK-MOW Bouvet

[3] Shao Yong was a philosopher of the Song Dynasty, *Song Chao* (960-1279), who greatly influenced the development of Neo-Confucianism in China. He studied the *Yijing* and did so following the *Xiangshu* Study (image-number study), an approach based on iconographic and cosmological concepts.

[4] Fuxi is a character from Chinese mythology and a civilizing hero.



DUO RERUM PRINCIPIA.

Perfectum. Imperfectum.

Yam. *Yn.*

Quatuor Imagines ex duobus Principiis proximè natæ.

Majus Perfectum. Minus Imperfectum. Minus Perfectum. Majus Imperfectum.

Tai-yam. *Xao-yn.* *Xao yam.* *Tai-yn.*

Octo Figuræ ex quatuor Imaginibus promanantes.

Kien. Cælum. *Tui.* Aquæ montium. *Li.* Ignis. *Chin.* Tonitrua. *Siven.* Venti. *Can.* Aqua. *Ken.* Montes. *Quen.* Terra.

1 2 3 4 5 6 7 8

Has octo Figuras, ex quibus quatuor ad perfectum, quatuor ad Imperfectum pertinent, in Orbem quoque describunt, cum mutuò inter sese, nec non vario, ad quatuor Mundi Cardines aspectu: Quibus etiam Cardinibus quatuor Zodiaci puncta, Solstitialia scilicet, & Æquinoctialia, quibus dum media rursus jungunt, octo Zodiaci quoque puncta & quasi Mundi Cardines describunt. Figuras interim sic describunt, ut à capite primoque numero semicirculum quatuor constantem Figuris ac numeris, producant ad lævam; & mox alterum à capite (seu quinto numero) rursus orsi cum totidem numeris ac Figuris ad dexteram describant, Orbemque totum conficiant, hoc modo.

De harum mutua vel conjunctione, vel oppositione *Confucius* ad Librum *Ye kim* (interprete *Cham Co-lao*) exponens: *Cælum* (inquit) & *Terra certam* ac determinatam *sedem locumque obtinent*, hæc inferiorem, illud superiorem. *Montes* & *Aqua montium* se *mutuo penetrant per humores* tam eos, qui in vapores resoluti ascendunt ex aquis locisque subterraneis (unde nubes & pluviæ) quam eos, qui densiores cum sint, descendunt; ex quibus deinde fontes existunt, & fluenta, & stagna. *Tonitrua* seu exhalationes siccæ & calidæ, & *Venti* seu vapores & halitus ventosi frigusque circumstans, *se mutuò comprimunt atque urgent*, necnon unita permixtaque se mutuo fovent ac juvant: Quare & Tonitrua majori cum impetu deorsum ruunt, & venti vicissim ab ignitis exhalationibus magis ac magis exæstuant. *Aqua* similiter, & *Ignis*, dum hic illius frigus, illa vicissim hujus ardorem temperat, adeo *non inter se pugnant* seque mutuo destruunt, ut contra cum insigni quodam emolumento rerum omnium socientur identidem, ac misceantur: Planè ut perspicuum sit, *octo Figuris* sic aliis alias opponi, ut non tam censeri possit oppositio rerum contrariarum, quàm earum, quæ, vel permisceantur, vel *sibi mutuo succedant atque* opitulentur, amica societas.

Figure 2– The trigrams according to the Fuxi or *Xiantian* order (Couplet, 1687, 42). Source: Staatliche Bibliothek Regensburg, shelfmark 999/2Philos.3043.

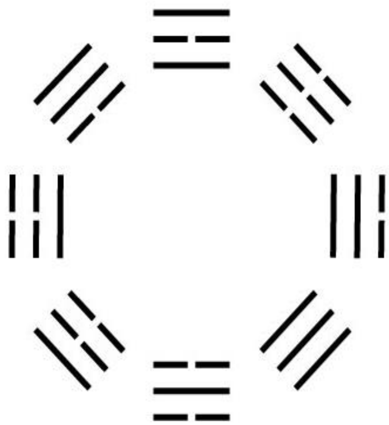


Figure 3 - The trigrams according to the Wen Wang or *Houtian* order. (Diagram created by the author).

The author chooses not to translate the two concepts but rather to explain them in the light of Shao Yong's philosophy (Shao Yong, 2004). As Anne D. Birdwhistell describes (1989), Shao Yong intended to explain the changes in the universe. He chose to do this through a cosmology from *Yijing* and whose study was called "the study of images and numbers," or *Xiangshu*. This domain is that of a theoretical order and abstract structure which systematizes the world and which differs from a particular experience by its level of abstraction. *Xiantian* and *Houtian* are two kinds of reality: theory and experience. Shao was interested in the theoretical aspects of reality, or the *Xiantian* (Birdwhistell, 1989, 66). The two realms of *Xiantian* and *Houtian* exist simultaneously and are two different levels of reality (Birdwhistell, 1989, 83). In other words, "[t]hey are two different aspects of the universe: *hou-t'ien* refers to the realm of a particular experience, and *hsien-t'ien* to the realm of the abstract patterns of structure, which particular experience follows" (Birdwhistell, 1989, 83). The order of the eight trigrams that organizes the sixty-four hexagrams in question in this article is the order of enumeration associated with Fuxi, that is *qian* ☰, *dui* ☱, *li* ☲, *zhen* ☳, *xun* ☴, *kan* ☵, *gen* ☶ and *kun* ☷ (see Fig. 2). The one associated with King Wen is seen in Fig. 3.

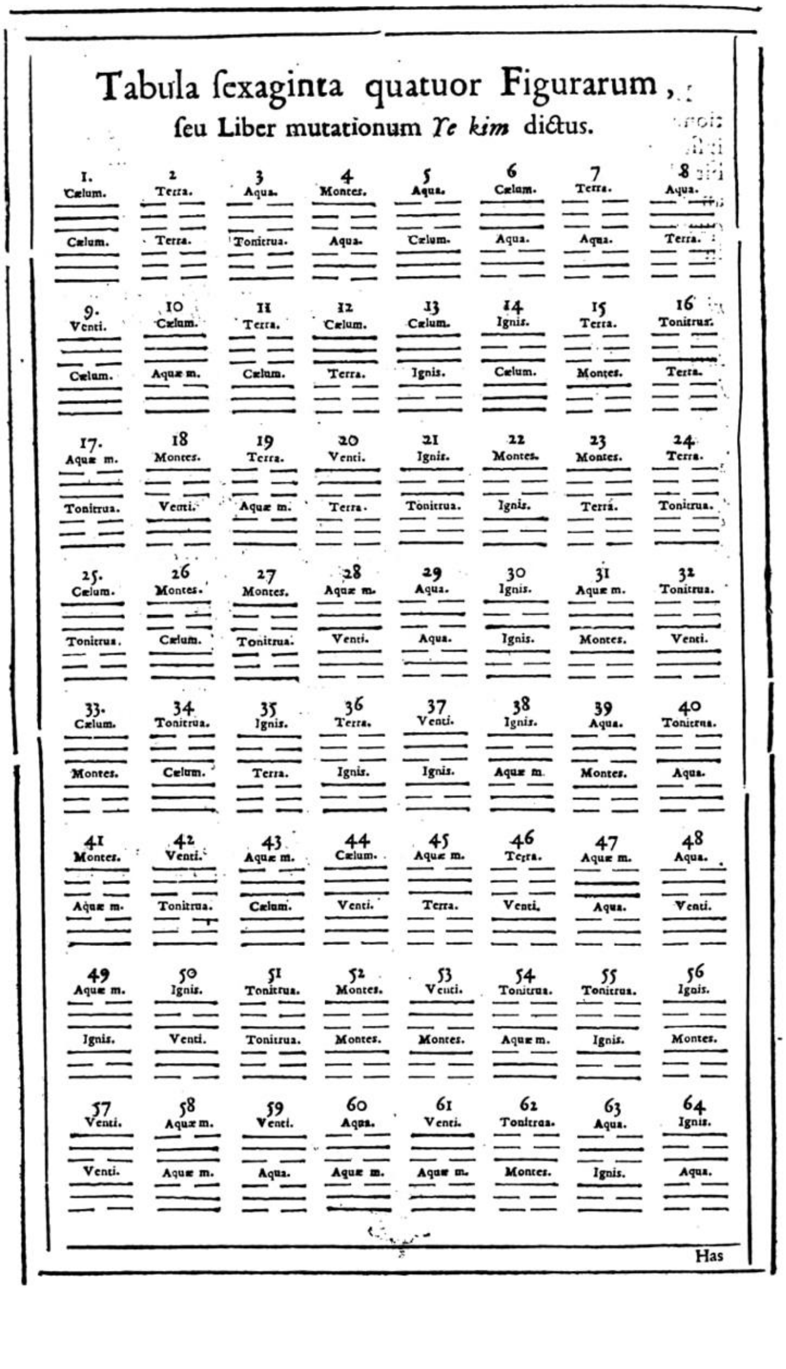

Tabula ſexaginta quatuor Figurarum,
ſeu Liber mutationum Ye kim dictus.
1. Cælum. Cælum.
2 Terra. Terra.
3 Aqua. Tonitrua.
4 Montes. Aqua.
5 Aqua. Cælum.
6 Cælum. Aqua.
7 Terra. Aqua.
8 Aqua. Terra.
9. Venti. Cælum.
10 Cælum. Aquæ m.
11 Terra. Cælum.
12 Cælum. Terra.
13 Cælum. Ignis.
14 Ignis. Cælum.
15 Terra. Montes.
16 Tonitrua. Terra.
17. Aquæ m. Tonitrua.
18 Montes. Venti.
19 Terra. Aquæ m.
20 Venti. Terra.
21 Ignis. Tonitrua.
22 Montes. Ignis.
23 Montes. Terra.
24 Terra. Tonitrua.
25. Cælum. Tonitrua.
26 Montes. Cælum.
27 Montes. Tonitrua.
28 Aquæ m. Venti.
29 Aqua. Aqua.
30 Ignis. Ignis.
31 Aquæ m. Montes.
32 Tonitrua. Venti.
33. Cælum. Montes.
34 Tonitrua. Cælum.
35 Ignis. Terra.
36 Terra. Ignis.
37 Venti. Ignis.
38 Ignis. Aquæ m.
39 Aqua. Montes.
40 Tonitrua. Aqua.
41 Montes. Aquæ m.
42 Venti. Tonitrua.
43 Aquæ m. Cælum.
44 Cælum. Venti.
45 Aquæ m. Terra.
46 Terra. Venti.
47 Aquæ m. Aqua.
48 Aqua. Venti.
49 Aquæ m. Ignis.
50 Ignis. Venti.
51 Tonitrua. Tonitrua.
52 Montes. Montes.
53 Venti. Montes.
54 Tonitrua. Aquæ m.
55 Tonitrua. Ignis.
56 Ignis. Montes.
57 Venti. Venti.
58 Aquæ m. Aquæ m.
59 Venti. Aqua.
60 Aqua. Aquæ m.
61 Venti. Aquæ m.
62 Tonitrua. Montes.
63 Aqua. Ignis.
64 Ignis. Aqua.
Has

Figure 4- Representation of the 64 hexagrams according to the Wen Wang or *Houtian* order (Couplet, 1687, 44).

While exchanging letters with Gottfried Wilhelm Leibniz (1646-1716), the latter explained his binary arithmetic, or dyadic, whose first written record is the *De Progressione Dyadica* dated 15 March 1679. It is a binary number system for noting all numbers with only two digits¾1, and 0¾that are used instead of ten as the basis of a scoring scale. The missionary then discovered an analogy between this new calculation and the diagram of the *Xiantiantu*. When one replaces 0 and 1 with the broken lines of the yin and the unbroken lines of the yang, the two systems are analogs and in harmony (see Fig. 5).

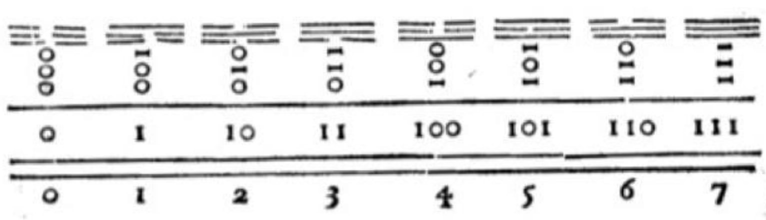


Figure 5 – Correspondence between the *bagua* and the binary arithmetic of Leibniz (E, 88). Source: https://hal.archives-ouvertes.fr/ads-00104781

This analogy made by Bouvet and validated by Leibniz indicates that the *Xiantiantu* could be considered binary. Researchers have discussed whether the diagram is binary or not.,[5] The author argues that *Xiantiantu* is binary, but that is not the purpose of this article. (Author, 2020) The *Xiantiantu* could be considered binary matters because it has implications in the history of binary arithmetic since it could be seen as an early form of binary arithmetic practice. Some scholars argue that Leibniz developed his dyadic influence by the *Xiantiantu.*[6] For example, Hu Yang and Li Zhangduo maintain this influence thesis. First, Leibniz saw the eight trigrams according to the Fuxi order and the sixty-four hexagrams based on the Wen Wang or *Houtian* order (Fig. 4) in the *Confucius sinarum philosophicus* edited by the Jesuit Philipp Couplet in 1687. Second, sources, which Leibniz had read, already qualified the diagram as "binary" before Father Bouvet did it. Third, Leibniz would have seen the diagram before 1679 because it had already been published (flipped, see Fig. 6) in Europe in the Jesuit Martino Martini's *Sinicae historiae dicas prima* (1658) (Hu & Li, 2006, 36-66).

[5] On this topic, see Sypniewski, 2000, 287-314; Shi Zhonglian, 2000; Zhao Zhongguo, 2008, 75-82; Zhang Yuanshan, 2016, 22-58; Mungello, 1977; McKenna & Mair, 1979, 421-41; and Li Jiemei, 1989, 39-42.

[6] See the remarks of Shi, 2000, 165; and Sun, 1999a, 329.

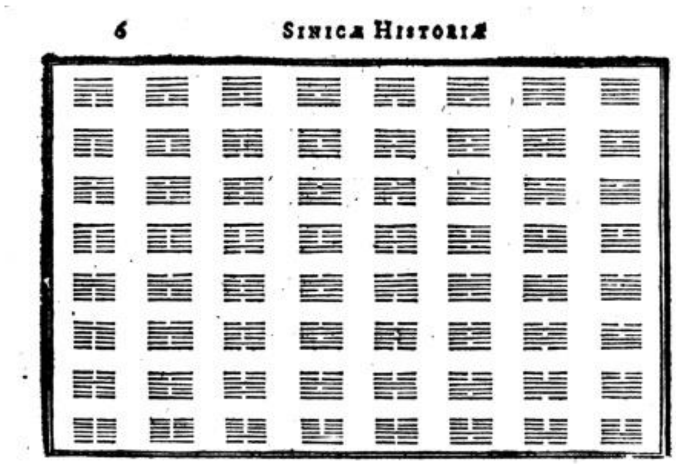


Figure 6 – The 64 hexagrams of the square diagram of the *Xiantiantu* printed flipped (Martini, 1658, 6). Source: Original from the Bavarian State Library, digitized by Google on 31 March 2015.

This idea of Leibniz being influenced by the *Xiantiantu* could also be found in sinological works such as A. D. Birdwhistell (1989, 75), and vulgarization articles (Von Aue, 2018).

However, some scholars opposed some counterarguments to this influence thesis. They contend that Leibniz had already developed his binary arithmetic before receiving the representation of the *Xiantiantu* from Joachim Bouvet in 1703. Sun Xiaoli (1999a, 239-43), Eric J. Aiton (1985, 245-48), and Jean-Pascal Alcantara (2006, 14), for instance, list the facts chronologically and argue that the philosopher invented his calculus before receiving the *Xiantiantu* from the Jesuit in 1703 since he had already written the *De Progressione Dyadica.* Nevertheless, these arguments fail to take into account the idea raised by Hu Yang and Li Zhangduo that Leibniz would have seen the diagram before 1679 in Martino Martini's book that Leibniz mentioned after 1675 (A IV, I, pp. 569-70).[7] They only focus on the *Xiantiantu* that the philosopher received in 1703 from Joachim Bouvet.

The objective of this article is to take these three arguments of Hu and Li into account since they create doubt regarding the influence of Leibniz, especially the third one stressing the publication of the diagram (flipped) in Europe in Martino Martini's book. The counterarguments of the scholars mentioned above are not sufficient to eliminate this doubt, and Hu and Li support the persistence of this influence thesis.

[7] As Franklin Perkins (2004, 108) points out, Leibniz frequently referred to Martini's book on China in 1675.

Regarding these contradictory positions and gaps in the treatment of this issue, the author aims to investigate if Leibniz was influenced by the *Xiantiantu* he saw before 1679 when he developed his binary arithmetic. Is Shao Yong's *Xiantiantu* the source of inspiration for Leibniz's dyadic? Or did he develop his binary calculus independently of the *Xiantiantu*, which only enabled him to publish his calculation as commonly accepted?

The author examines this issue by establishing the chronology exposing the development of binary arithmetic by Leibniz where I insert his exposure to the *Yijing*, the trigrams and hexagrams, the *Xiantiantu* and the *Houtiantu,* in other words, the *bagua* system in general. Then, through a review of the literature, the author gathers the arguments asserting the influence of the *Xiantiantu* on Leibniz' development of dyadic and how the The author examines this issue by establishing the chronology exposing the development of binary arithmetic by Leibniz, where the author inserts his exposure to the *Yijing*, the trigrams and hexagrams, the *Xiantiantu,* and the *Houtiantu,* in other words, the *Bagua* system in general. Then, through a review of the literature, the author gathers the arguments asserting the influence of the *Xiantiantu* on Leibniz's development of dyadic and how the scholars responded to this thesis of influence. Finally, the author extends the point of view on this issue by bringing in other areas of Leibniz's philosophy related to binary arithmetic, something that no scholar mentioned in the treatment of this problem.

This thesis is that Leibniz was not influenced by the *Xiantiantu* sent by Joachim Bouvet in 1703 nor by the diagram published flipped in the book of Martino Martini that Leibniz mentioned after 1675. The author's findings show no influence of the *Xiantiantu* in the creative development of binary arithmetic, and the impact of the diagram is only the publication of the *Explication de l'Arithmétique binaire* in 1703. The author argues that knowing the *Xiantiantu* as a form of binary arithmetic would have changed his view on Chinese writing since 1675 and, therefore, his development of Universal characteristics. Indeed, Leibniz intended to create a Universal Characteristic, that is to say, a universal formal language designed to eliminate the ambiguity and fluctuation of natural language, reducing itself to an "arithmetic calculation" to judge controversies and invent a new knowledge (Antognazza, 2009, 92).[8] Leibniz thought about using Chinese writing as signs for developing his Universal Characteristic, whose perfect form was binary arithmetic. However, he discarded this possibility in 1679 since they were not combinatory or useful for reasoning until 1698, when Joachim Bouvet introduced the characters of the *Yijing* to him. The analogy between his binary arithmetic and the *Xiantiantu* made the philosopher think he had found a rational script that could play a role in his project. It also brought back Chinese writing as a potential sign in the Universal Characteristic since the Jesuit genealogically linked it with the hexagrams of the *Yijing.* Father Bouvet believed Fuxi was the first to have transmitted to humankind not only the

[8] On Leibniz' Universal Characteristic, see Couturat, 1901, 81-118; Kikai, 1983, 374-83; Widmaier, 1983; Perkins, 2004, 140-45; Pombo, 1987; Rossi, 1993, 201-17; Eco, 1997, 307-21.

system of the eight trigrams but also the first Chinese writing since he presented the *Bagua* as the origin of Chinese scripture (Bouvet to Leibniz, 4 November 1701; A I, 20: N 318, 538; CB Letter I, 10; WB, 338-40). It implies that if Leibniz was aware of the *Xiantiantu* as a form of binary arithmetic, he would have used it and, by extension, Chinese script (with the genealogical link between the *Bagua* and Chinese writing) assigned for his Universal Characteristic. It would have changed his view on Chinese writing. However, the author's findings do not show that change or use.

This contribution aims to clarify the issue of the influence of the *Xiantiantu* on the development of binary arithmetic. The author brings the original argument of Leibniz's view on Chinese writing and, consequently, the Universal Characteristic project to the debate. It connects binary arithmetic, Universal characteristics, the *Bagua*, and Chinese writing. This lost connection between the four domains in Leibniz's philosophy matters since it brings a strong argument into the debate to seriously counter this enduring idea of influence. It will contribute to a change in the way we think about the Leibnizian development of binary arithmetic, linked to the *Xiantiantu*, not as an influence on the creative process but as a promotion of the *Explication* of 1703.

To address this issue, the chronology of the facts concerning the development of binary arithmetic and Leibniz's exposure to the *Bagua*, in general, will first be exposed. Then, the arguments of the researchers questioning it will be closely examined and their counterarguments. Finally, it will be shown that Leibniz was not influenced by the *Xiantiantu* when developing the binary arithmetic because if this were the case, it would have changed his view on Chinese writing and, consequently, his research on the Universal Characteristics.

I. Chronology of the Development of Binary Arithmetic by Leibniz and His exposure to the *Bagua* System

In order to clarify if the *Xiantiantu* influenced Leibniz during the development of binary arithmetic, it is first necessary to establish a chronology of the events, that is to say, the development of binary arithmetic by Leibniz. The author will insert the exposure of the philosopher to the *Yijing*, the trigrams or hexagrams, the publication of the diagrams of *Xiantiantu* and *Houtian*.[9] The author will mention his changing views on Chinese script in the last part of the article.

First, Leibniz was not the only one to have worked on a binary system. Thomas Harriot (1560-1621) had already drawn up a table of binary values (but he had not noted it in figures) in 1600. Juan Caramuel Y Lobkowitz (1606-1682) *in Mathesis Biceps* (1670) studied number systems with other than ten bases where one finds a few pages on the binary system. Then Jean Neper (1550-1617) developed the science of calculation by using sticks in his treatise *Rhabdologie* (1617). Francis Bacon also

[9] The author of this article relies on the works of Knobloch, 2018, 225-46; and Glaser, 1971.

developed a system based on two letters *De numeris multiplicibus ex sola characterum numericorum additione agnoscendis* (1665), and Pascal described a system of duodecimal numbers (Zacher, 1973, 9-56; Knobloch, 2018, 242-46).

The *Xiantiantu* was first published in Europe in 1658 by the Jesuit Martino Martini (1614-1661) in his *Sinicae historiae dicas prima*. Leibniz connected for the first time the Chinese writing and his Universal Characteristic project in his *De Arte combinatoria* (1666) (GM V, 50; Widmaier, 1983, 132). According to Leibniz's own remarks, he started to develop his binary calculation during his stay in Paris (March 1672-January 1673) (Knobloch, 2018, 243). In 1675, Leibniz referred to the book of Martino Martini. He was also in contact with the orientalist Gottlieb Spitzel (1639-1691), who published the *De re literaria Sinensium* (1660), following the information of Martini (1658) in the crafting of his book. Leibniz had an exchange of twelve letters with Spitzel between 1669 and 1672. A presumed manuscript written in October 1676 mentions the dyadic and the duodecimal system (Manuscript LH XXXV 15,4 sheet 7, see Knobloch, 2018, 243).

The first dated manuscript containing the dyadic is the *De Progressione Dyadica* of 15 March 1679 (Serra, 2010; Alcantara, 2006, 14; Couturat, 1901, 473-78; C, 574). The first part of this document describes the binary notation and indicates how to pass from one binary number to the next. According to Mattia Brancato (2021, 4), it "is followed by an obscure second part in which Leibniz shifts his attention from arithmetic to algebra." It helps to understand "the mathematical importance of the binary system in Leibniz's mind, the reason why he developed it and the connection between its mathematical relevance and its philosophical relevance." Leibniz then explained his binary calculation in the *Summum calculi analytici fastigium per calculum algoritmicum* of December 1679, the second version of *De progressione dyadica*, highlighting the fruitful nature of his binary calculation (Knobloch, 2018, 243).

In 1687, Leibniz knew the publication *Confucius sinarum philosophicus* edited by Couplet that he mentioned the same year (A I, 4, 622). After two manuscripts on the dyadic, *Mira numerorum omnium expressio per 1 et 0* of 17 May 1696 (Zacher, 1974, 225-28); and *Wunderbarer Urprung aller zahlen aus 1 und 0* of 18 May 1696 (Zacher, 1974, 229-34); on 2 January 1697, Leibniz subsequently wrote a letter to Duke of Brunswick-Lüneburg-Wolfenbüttel. He introduced his binary calculation represented on the sketch of a medallion bearing the image of the Duke (see Fig. 7) (Ching & Oxtoby, 1992, 72).[10]

[10] It seems he never sent this letter.

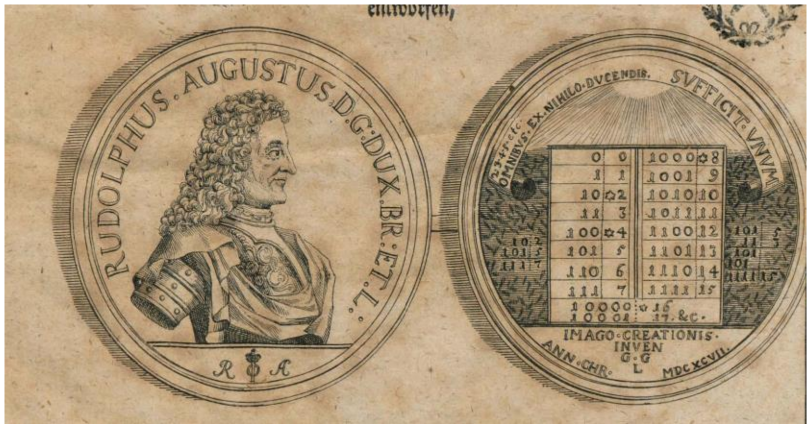


Figure 7 - Medallion offered by Leibniz to Duke Brunswick-Lüneburg-Wolfenbüttel (Nolte 1734). Source: University Library of Tubingen. Bb

In mid-January early February 1697, Leibniz exposed his new calculation to the Jesuit Claudio Filippo Grimaldi (1638-1712) (A I, 13, N. 321, 518-22; WB, 74-103). In the letter dated 28 February 1698, Joachim Bouvet mentioned for the first time, "[…] the first characters of this nation composed of small whole and divided horizontal lines, whose invention they attribute to Fuxi […]" (A I, 15, N 238, 355; CB Letter E, 10; WB, 170). In two letters on 29 March 1698 (A II, 3, N 165, 425-28) and 17 May 1698 (A II, 3, N 175, 449-53), Leibniz exchanged two letters on binary arithmetic with Johann Christian Schulenburg (1668-1732).

Leibniz exposed to Bouvet the principles of his binary arithmetic in the letter of 15 February 1701 (A I, 19, N 202, 401-15; CB Letter H; WB, 300-25). On 26 February 1701, he sent an article on the dyadic entitled *Essay d'une nouvelle Science des nombres* to Bernard Le Bouyer (or Le Bovier) de Fontenelle, perpetual secretary of the Royal Academy of Sciences in Paris (Zacher, 1974, 250-61). However, the philosopher asked Fontenelle not to publish his article and wait until he found "better samples" (12 July 1702, FC, 209). Meanwhile, he exchanged letters that mentioned the binary system with the Swiss mathematician Jean Bernoulli (1667-1748) from 5 April to 7 May 1701 (GM III 1, 656-69).

Joachim Bouvet informed Leibniz of his analogy between the hexagrams of the *Xiantiantu* and the dyadic in a letter dated 4 November 1701. He attached the *Xiantiantu* composed of the round and square diagrams of the hexagrams drawn up by Shao Yong (see Fig. 1) (A I, 20: N 319, 556, CB Letter I; WB, 330-77). Leibniz did not receive this letter until 1 April 1703. On 7 April 1703, he sent a new version of his article to de Fontenelle (FC, 225). On 5 May 1703, the *Explication de l'Arithmétique binaire*, his only publication on binary arithmetic, appeared in Paris's *Journal de l'Académie royale des sciences*.

II. Arguments and Counterarguments Regarding the Influence of the *Xiantiantu* on the Development of Binary Arithmetic by Leibniz

1. Arguments stating the influence thesis

Having drawn up the chronology concerning the development of binary arithmetic by Leibniz and his exposure to the diagram, the author will analyze the arguments of some researchers who argue for the influence thesis.

The reasoning of the researchers Hu Yang and Li Zhangduo is that if there is an analogy between Leibniz's dyadic and the *Xiantiantu* of Shao Yong, then this diagram is binary. It, therefore, represents a form of binary arithmetic and is even the origin of the binary system. They deduce ultimately that Leibniz did not invent binary arithmetic. He developed it being influenced by the *Xiantiantu*.[11] The proof is that he saw the diagram before inventing it in 1679 since it had already been published in Europe before the Jesuits provided it.

The demonstration of Hu Yang and Li Zhangduo is based on three arguments. The first argument is that Leibniz was not only aware of the order and orientation of the eight trigrams of Fuxi but also of the arrangement of the sixty-four hexagrams according to the order of Wen Wang through his reading of *Confucius sinarum philosophicus* by Couplet published in 1687 (Hu and Li, 2006, 23; Hu & Li, 2004, 67). This argument is valid since Leibniz read this work. However, if he had these three elements in front of him, he did not see the diagram of the sixty-four hexagrams according to the order of Fuxi, the *Xiantiantu*.

The second argument relies on the fact that Gottlieb Spitzel would have used the term "*binarium*" to qualify the diagram of the *Yijing* in his *De re literaria Sinensium* (1660). Indeed, Gottlieb Spitzel introduced the generation principle of the universe, coming from yin and yang, divided into four figures and eight trigrams, resulting from a mathematical operation of multiplication of two squared (Spitzel, 1660,166). This derivation from the yin and yang dyad is a "principle of binary multiplication" (*principiis per binarium multiplicatis*) which means for Hu and Li that he already considered the principle of the *Yijing* as binary (Hu & Li, 2006, 43; Hu & Li, 2004, 68).

Regarding this argument, since Leibniz also read *De re literaria Sinensium* and exchanged correspondence with Spitzel, this idea is plausible. However, the adjective *binaries* mean "double," or "two" in Latin. Thus, Spitzel only says that the two principles are multiplied by two (*Ex duobus itaque illis principiis per binarium multiplicatis)*. When he writes the word "double," the meaning may not be a binary system such as the one developed by Leibniz.

Finally, the third argument lies in the fact that the diagram of the *Xiantiantu* had already been published in Martino Martini's work in 1658, but flipped[12] (Von Collani,

[11] This argument is taken up more nuanced by Zhu & Zhu, 2011, 22.

[12] Fig. 6 indeed shows that, if we compare it to Fig. 1, the diagram is flipped due to an error

1996, 235). Leibniz had read this book, and he referred to it in 1675, before he was in contact with the version of the diagram that Father Bouvet sent him in 1703 (Hu & Li, 2006, 64-66; Hu & Li, 2004, 70).

Thus, these three arguments lead, according to them, to a new understanding of the binary arithmetic invented by Leibniz, which turns out to be a system derived from the diagram of the *Bagua* of Fuxi, in other words, the *Xiantiantu* (Hu & Li, 2006, 36). They conclude that Leibniz does not have priority over the so-called "invention"; he deduced it from the research of his predecessors. Finally, China's binary system comes from (Hu & Li, 2006, 114, 122, 130).

The author's analysis is that having seen Shao Yong's diagram in Martini's book is a strong argument, and the other two fail to prove anything. Although this argument does not prove that the Xiantiantu influenced Leibniz, it nevertheless sows doubt, and it calls for a clarification which the author will now develop.

2. Existing counterarguments in reaction to the influence thesis

Counterarguments to this re-examination of the development of the dyadic by Leibniz, which the author gathered into five ideas, exist in the secondary literature.

The author of this article will start with Leibniz himself, who, in his letter of 18 May 1703 to Joachim Bouvet, claimed that if he had not invented his binary calculation, he would observe the hexagrams for a long time without understanding them. He then specified that he invented this system more than twenty years ago, that is to say, before 1683, and that he was waiting to demonstrate its great uses before publishing it (A I, 22, N 218, 353; CB Letter J, 7-8; WB, 404).

The first of the counterarguments I identified is to list the facts chronologically. However, without mentioning the contacts of Leibniz with the *Bagua* system, such as Sun Xiaoli (Sun 2006, 124 & 1999a, 239-43).[13] Jean-Pascal Alcantara (2006, 14) also objects that Leibniz invented "dyadic calculus by taking cognizance of the table of hexagrams dispatched by Bouvet, contrary to what the Chinese most often believe." He denies the idea that Leibniz could have been influenced by the *Xiantiantu* sent by the Jesuit since the facts show that he had already written the *De Progressione Dyadica* in 1679. Li Jiemei (1989, 42), Wang Yusheng (1997, 84), Eric J. Aiton (1985, 245-48) does the same.

The second counterargument is that the philosopher did not invent the binary system alone, as the author already mentioned in the chronology part. Jean-Pascal Alcantara specifies that "it is not true that Leibniz invented the dyadic calculus on his own, preceded by Harriot and Pascal" (Alcantara, 2006, 14).

The third counterargument is to deny that the *Xiantiantu* is binary since if it is not binary, Leibniz is the one who "invented" binary arithmetic. As seen above, the controversy was launched by Arthur Waley (1921). Against Waley's words, Paul

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during the manufacture of the printing plate (Hu & Li, 2006, 64).

[13] Li (1989, 41-42) does this but belongs to the third kind of counterargument (denying the *Xiantiantu* is binary).

Pelliot (1878-1945) confided that he did not think that the classification of the *Xiantian* was a binary notation. The reason for his assertion is that "[…] it would be very extraordinary if the Chinese of "3000 years before our era" had known the zero and the rule of position" (Pelliot, 1922, 90). He points out that it is not necessary to have the mathematical knowledge to create the Shao Yong system; it is just a matter of combining the lines between them (Pelliot, 1922, 90-91; McKenna & Mair, 1979, 428).

Joseph Needham (1956, vol. 2, note 41, 342), Marcel Granet (1934), and René Étiemble (1988, tome 1, 406) maintain the same idea, as well as that of the absence of mathematical operation in the *Yijing* or the Chinese Classics. Sun Xiaoli supports the same (Sun, 1999b, 57; Sun, 2006, 245) and further notices that yin and yang do not have the functions of 0 and 1 (Sun, 2006, 132). Finally, Li Jiemei and Shi Zhonglian (2000, 166) subscribe to the same argument and add that the methodology used in the *Xiantiantu* is different from the one used in binary arithmetic, that is to say, "one divided by two (and not one bit every two)" (Li, 1989, 39-41).

The fourth argument is supported by Zhao Zhongguo, who maintains that the analysis of the *Xiantiantu* established by Leibniz (with Father Bouvet) is only an interpretation of the philosopher "[…] in his own mathematical field of vision, which shows his intelligence, his mathematical awareness, and his knowledge" (Zhao, 2008, 79). He concludes that the *Xiantiantu* "does not provide him with a ready-to-use binary system" (Zhao, 2008, 79).

Finally, the fifth and last argument is that the impact of the *Yijing* on binary arithmetic was only its role in the publication of the *Explication* in 1703. Zhu Xinchun and Shi Yumin specify that the influence of the *Xiantiantu* on the Leibniz dyadic is to "contribute to the publication and public dissemination of binary arithmetic, and this influence obviously does not belong to the creative level" (Zhu & Shi, 2010, 92). Of course, this influence does not rise to the level of the creation of the binary system. Sun Xiaoli (1999a, 245) and Zhu Xinchun and Zhu Guangyao (2011, 22) take up this idea.

My conclusion is that none of the five arguments mentioned is able to neutralize the idea that Leibniz would have seen the *Xiantiantu* before 1679 because they simply did not take it into account. Likewise, to say that the arguments of Hu and Li prove nothing is not enough. This is why I will expose now an original argument in order to reaffirm that Leibniz developed binary arithmetic independently of the *Xiantiantu*.

III. Original Argument Invading the Influence of the *Xiantiantu* on the Development of Binary Arithmetic by Leibniz

The author of this article argues that Leibniz was not influenced by the *Xiantiantu* sent by Joachim Bouvet in 1703 nor by the diagram published (flipped) in the book of Martini. It would have changed his perspective on Chinese writing and his research on Universal Characteristics.

The Universal Characteristic that Leibniz intended to create aimed to reason like a calculation in the broader sense than a numerical calculation because it is extended to the logical calculation, which relates to ideas. His project was one of a language capable of functioning as a *calculus philosophicus* in the way an arithmetic system does. The reasoning is applied with combinatorial rules. They are those of the art of combinations or the science of forms, in other words, the Characteristics.

The philosopher foresaw the development of his Universal Characteristic in several stages, from his youth work, *Dissertatio de Arte Combinatoria* (1666). It involved 1) the systematic identification of all simple/primitive concepts into an alphabet of human thoughts (the General Encyclopaedia); 2) the choice of signs to symbolically represent these simple ideas with a fixed, unequivocal, and unambiguous correlation; and 3) the development of a combinatorial method which governs the combination of these primitive concepts which can also be represented symbolically, in order to create complex ideas (Antognazza, 2009, 434; Rossi, 1993, 202*;* Eco, 1997, 308; Couturat, 1901, 48-50).

When searching for signs of his Universal Characteristic (step 2), Leibniz became interested in Chinese writing. The actual characters were among the signs that the philosopher was looking for in his project.[14] As a reader of Francis Bacon, Leibniz was familiar with this notion of conventional signs, which directly signify an idea without the means of speech. It was universal since it was readable by everyone in their own language. It corresponded to two of the criteria that the signs of the Universal Characteristic had to satisfy (Perkins, 2004, 142).

According to Bacon, Chinese writing was made up of real characters (Bacon, 1605 (1803), 146-47). Through Bacon, Leibniz understood that it would be possible to write a philosophical language with these real characters. The philosopher, therefore, sought real and philosophical characters for his Universal Characteristic, which should allow the representation of ideas and the analysis of thoughts (Widmaier,1983, 17). Leibniz consequently became interested in Chinese characters, which, in his view, directly represented ideas and were universal.

Leibniz's representations changed[15] and it is possible to identify four periods. [16] During the first period from 1666 to 1679, Leibniz thought Chinese characters as actual characters, hieroglyphics, pictographic and sacred writing (Widmaier, 1983, 133). They are also ideographic, that is to say, more pictographic than phonetic (GM V, 50). However, from 1679, they were, in his eyes, neither combinatorial nor useful for reasoning (GP VII, 204; Couturat, 1901, 81). He also considers them distant from

[14] There are other selection criteria for the signs of the Universal Characteristic of Leibniz that I do not have the space to develop here. See Perkins, 2004, 142; and Dascal, 1978.
[15] The terminology and view Leibniz used when he spoke about a "characteristica realis," "scriptura rationalis," was also changing. See Schneider, 1994, 213-236; Pombo, 1987, 123; Kikai, 1983, 374; Arndt, 1967, 71; Widmaier, 1983, 9.
[16] See the other divisions in Cook & Rosemont, 2000, 129; Hao, 2000, 188; Widmaier, 1983, 134. The author agrees with the periods that Rita Widmaier developed.

the analysis of thoughts (Leibniz to Herzog Johann Friedrich von Hannover, April 1679, A II I, 706-707); but also ambiguous (Dutens VI, part 2, 31; Leibniz to Herzog Johann Friedrich von Hannover, August 1685-October 1687, A II I, 876). He, therefore, stopped thinking about them for the Universal Characteristic.

Then in the second period, from 1679 to 1701, Leibniz learned of the works on the *Clavis Sinica*, or key to decipher the Chinese writing, which was the object of research of proto-sinologists, that is to say, the actors of the beginning of the study of China in Europe, like Andreas Müller (1630-1694) and Christian Mentzel (1622-1701) (A I, 2, 491-92; A I, 14, N. 445, 780-83; Mungello, 1989, 198-203). Leibniz corresponded in 1679 with the former and in 1697 with the latter. The *Clavis* gave him hope to find a missing rational structure from Chinese writing. In 1694, he thought the Chinese characters were international since they were used by people of different languages (Dutens VI, part. 2, 135). However, the fact that the Chinese language is particular pushed Leibniz to abandon Chinese writing as a potential sign of his Universal Characteristic. In a letter to Father Verjus dated 12 December 1697, his design for a universal language was independent of particular languages (Perkins, 2004, 143).

Between 1701 to 1707—the third period, his exchange with Joachim Bouvet gave him a positive view of Chinese writing. From the latter, Leibniz learned that the trigram system was the origin of Chinese writing since Fuxi was the inventor of the trigrams. The combination of the trigrams gave the hexagrams system of the *Xiantiantu*, which was rational since it was a form of binary arithmetic. The analogy established by Father Bouvet between the dyadic and the *Xiantiantu* proved that the Chinese had developed a combinatorial sort of characteristic independently and that, moreover, it was reducible to numerical analysis as he conceived it for his characteristic (Antognazza, 2009, 436). The hexagrams system was, therefore, a form of binary arithmetic. The dyadic being for the philosopher, the perfect form of the Universal Characteristic (C, 284; and Shi, 2000, 166), the system of hexagrams could be used to form a new characteristic.[17] Leibniz then intended to use the symbols of the *Yijing* to establish a new characteristic (Leibniz to Bouvet, 28 July 1704; A I, 23: N. 422, 578; CB Letter L, 1; WB, 456). The author agrees with David E. Mungello, who argues that Father Bouvet influenced Leibniz's change of view on Chinese writing (Mungello, *Curious Land*, 1989, 204). Indeed, after having learned about the *Yijing* hexagrams through this genealogical with Chinese writing, Leibniz considered them less pictographic and more philosophical (Leibniz to Bouvet, 18 May 1703; A I, 22: N 218, 362; CB Letter J, 27; WB, 422-24).

Finally, during the fourth and last period, from 1707 to 1716, Leibniz was more careful, even ambivalent, but he did not forget the Chinese script. In 1707, the

[17] Bouvet initiated this possibility (Bouvet to Leibniz, 4 November 1701, A I, 20: N 318, 538, CB Letter I, 11; WB, 340) and Leibniz seems won over by this idea (see Leibniz to Bouvet, 15 February 1701; A I, 19: N 202, 412; CB Letter H, 34-35; WB, 316; and 18 May 1703; A I, 22: N 218, 357; CB Letter J, 18; WB, 414).

philosopher had learned from Father Augustin Cima that the Chinese characters were composed of fundamental characters (today called radicals), whose combinations formed all the others (Dutens V, 485). Leibniz believed these fundamental characteristics could be compared to simple concepts of the Universal Characteristic because they are directly linked to an idea (Leibniz to Veyssière la Croze, 8 October 1707, Dutens V, 484-85). However, beset with doubts, he was cautious, having no more news from the Jesuit from whom he awaited more information for his project (See E, 89; NE, 398; Leibniz to Bouvet, 13 December 1707; A I, 27: N. 38091; CB Letter O, 5; WB, 602).

Conclusion

From this chronology of the Leibnizian representations of Chinese writing and the potential use of the *Xiantiantu's* symbols in his Universal Characteristic project, the author deduces two elements that invalidate the idea by Hu Yang and Li Zhangduo of the influence of the *Xiantiantu* on the development of binary arithmetic by Leibniz. It proves that Leibniz developed his dyadic independently of the *Xiantiantu*. [18]

First, if Leibniz had an enlightened knowledge of the *Xiantiantu* as early as 1675 and his reading of Martini, it would have changed his view on Chinese writing, appearing in the four periods. These periods show that his view changed when he learned about the *Xiantiantu* by Joachim Bouvet from their correspondence starting in 1697. If he had known this diagram and understood that it was a form of binary arithmetic, then he would have also changed his perspective on Chinese writing as early as 1675 since the analogy between the two systems would have given a rational, mathematical and combinatorial aspect to Chinese writing through this genealogical link with the trigrams and the hexagrams. However, the chronology of his representations of Chinese writing does not show this change at this moment.

Second, if Leibniz had understood that the *Xiantiantu* contained a form of binary arithmetic, his research on the Universal Characteristic would not have taken the same path. Since the binary arithmetic was the perfect Characteristic for him and the *Bagua* system of the *Xiantian* was a form of binary arithmetic, it could be used as a

[18] On the contrary, a real influence, for instance, would be the one of the *De supputatione multitudinis a nullitate per unitates finitas in infinitum collineantis ad deum* (1679) of Erhard Weigel, the mathematics and metaphysics teacher of Leibniz at Jena in 1663. Couturat (1901, 473) and Grua (G, Tome I, 330) support that the influence of Weigel would be only on a mathematical level. However, Mattia Brancato shows that the influence of Weigel's *De Supputatione* "is much deeper and it has its roots also in the metaphysical background related to binary arithmetic" (Brancato 2016, 157). He argues that Weigel has influenced Leibniz from the beginning in mathematics in the terminology he used and the reference to the Pythagorean tradition. It was in his use of a different base model, his connexion with unity and nothingness with the arithmetical operation, and the homogeneity of the logical principles in a relational way to describe the world (Brancato 2016, 170-171).

sign of his Universal Characteristic. In addition, because of the genealogical link between the trigrams and the hexagrams with the Chinese writing, it is finally Chinese writing itself—with its rational origin—that could be used for his project. However, this change does not appear in the development of his project. These elements show that Leibniz was not influenced in his creation of binary arithmetic by the *Xiantiantu* published flipped in Martini's book after 1675, nor by the one sent by Joachim Bouvet in 1703. The only influence was to help with the publication of the *Explication de l'arithmétique binaire* in 1703.

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